CENTER OF MASS AND COLLISION

1. Two balls A and B moving in the same direction collide. The mass of B is p times that of A. Before the collision the velocity of A was q times that of B. After the collision A comes to rest. If e be the coefficient of restitution then which of the following conclusion/s is/are correct?

[NSEP-2017]

(A) $e=\frac{p+q}{pq-p}$ (B) $e=\frac{p+q}{pq+q}$ (C) $p\geq\frac{q}{q-2}$ (D) $p\geq1$

Ans. **(ACD)**

Sol.

$mqu+mpu=mpv_f$

$v_f=\frac{(q+p)u}{p}$

$e=\frac{(q+p)\frac{u}{p}}{qu-u}=\frac{q+p}{pq-p}$

$e\leq1$

$\frac{q+p}{pq-p}\leq1$

$p\geq\frac{q}{q-2}$

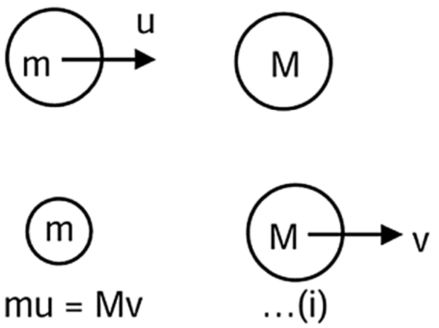
2. A ball of mass m hits directly another ball of mass M at rest and is brought to rest by the impact. One third of the kinetic energy of the ball is lost due to collision. The coefficient of restitutions is

[NSEP-2017]

(A) $1/3$ (B) $1/2$ (C) $2/3$ (D) $\sqrt{\frac{2}{3}}$

Ans. (C)

Sol.



$mu=Mv \quad ...(i)$

$\frac{p_f^2}{2M}=\frac{2}{3}\frac{p_i^2}{2\,m}$

$\frac{m}{M}=\frac{2}{3}=\frac{v}{u}=e$

3. Two particles A and B of equal masses have velocities $\vec{V}_A = 2\hat{i} + \hat{j}$ and $\vec{V}_B = -\hat{i} + 2\hat{j}$. The particles move with accelerations $\vec{a}_A = -4\hat{i} - \hat{j}$ and $\vec{a}_B = -2\hat{i} + 3\hat{j}$ respectively. The centre of mass of the two particles move along **[NSEP-2017]**

(A) a straight line
(B) a parabola
(C) a circle
(D) an ellipse

Ans. (B)

Sol. $\bar{v}_{cm} = \dfrac{\hat{i} + 3\hat{j}}{2}$ $\bar{a}_{cm} = \dfrac{-6\hat{i} + 2\hat{j}}{2}$

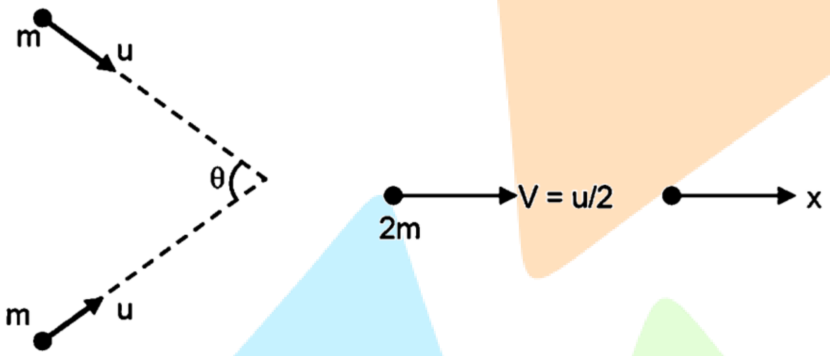
$= -3\hat{i} + \hat{j}$

4. Two bodies of equal masses moving with equal speeds make a perfectly inelastic collision. If the speed after the collision is reduced to half, the angle between their velocities of approach is **[NSEP-2018]**

(A) 30°
(B) 60°
(C) 90°
(D) 120°

Ans. (D)

Sol.



According to linear momentum conservation along x-axis

$2mu\cos\dfrac{\theta}{2} = 2m\dfrac{u}{2}$ $\therefore \cos\dfrac{\theta}{2} = \dfrac{1}{2}$

$\dfrac{\theta}{2} = 60$

Hence $\theta = 120^\circ$

5. Rocket fuel is capable of giving an exhaust velocity of $v_{rel} = 2.4\ km/s$ in the absence of any external forces. The fuel required per kg of the payload to provide an exhaust velocity of $12\ km/s$ to the rocket is: **[NSEP-2018]**

(A) 3670 kg
(B) 8000 kg
(C) 147.4 kg
(D) 478.4 kg

Ans. (C)

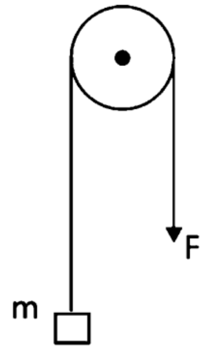
Sol. $v = v_{rel}\,\ell n\left(\dfrac{m_0}{m}\right) = 12\ km/s$

$\ln\left(\dfrac{m_0}{m}\right) = 5$

$m_0 = me^5 = 148.41\ m\ kg$

$m = 1\ kg,\ m_0 = 148.41\ kg$

6. A block of mass $m = 10$ kg is hanging over a frictionless light fixed pulley by an inextensible light rope. Initially the block is held at rest. The other end of the rope is now pulled by a constant force F in the vertically downward direction. The linear momentum of the block is seen to increase by 2 kg m/s in 1 s (in the first second). Therefore, **[NSEP-2019]**



(A) the tension in the rope is F

(B) the tension in the rope is 3 N

(C) the work done by the tension on the block, in first second, is $= 19.8$ J

(D) the work done against the force of gravity, in first second, is $= 9.8$ J

Ans. **(AD)**

Sol. $\Delta V = \dfrac{\Delta P}{m} = 0.2 \text{ m/s}$

at $= 0.2$

$a = 0.2 \text{ m/s}^2$

$s = \dfrac{1}{2}at^2 = 0.1$ m

$F - 98 = 10 \times 0.2$

F = 100

$w_g = -9.8$ J

W.D against gravity

+9.8 J

7. A ball of mass m_1 travels horizontally along the x-axis in the positive direction with an initial speed of v_0. It collides with another ball of mass m_2 that is originally at rest. After the collision, the ball of mass m_1 has velocity $\left(v_{1x}\hat{i} + v_{1y}\hat{j}\right)$ and the ball of mass m_2 has velocity $\left(v_{2x}\hat{i} + v_{2y}\hat{j}\right)$. Identify the CORRECT relationship(s) **[NSEP-2019]**

(A) $0 = m_1 v_{1x} + m_2 v_{2x}$

(B) $m_1 v_0 = m_1 v_{1y} + m_2 v_{2y}$

(C) $0 = m_1 v_{1y} + m_2 v_{2y}$

(D) $m_1 v_0 = m_1 v_{1x} + m_2 v_{2x}$

Ans. **(CD)**

Sol. $m_1 v_0 \hat{i} = m_1\left(v_{1x}\hat{i} + v_{1y}\hat{j}\right) + m_2\left(v_{2x}\hat{i} + v_{2y}\hat{j}\right)$

$m_1 v_1 y + m_2 v_2 y = 0$

$\& m_1 v_0 = m_1 v_1 x + m_2 v_2 x$

8. A body of mass 10 kg at rest explodes into two fragments of masses 3 kg and 7 kg. If the total kinetic energy of two pieces after explosion is 1680 J, the magnitude of their relative velocity in m/s after explosion is: **[NSEP-2020]**

(A) 40 (B) 50 (C) 70 (D) 80

Ans. (A)

Sol. Let the stationary mass m explodes in to m_1 and m_2

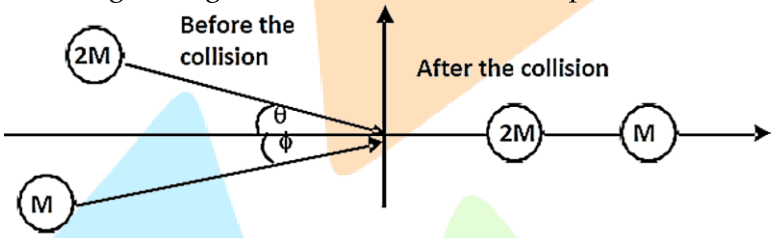
By conservation of momentum $m_1v_1 + m_2v_2 = 0$ (1) and Energy is

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = 1680 \Rightarrow \frac{1}{2}m_1v_1^2\left(1+\frac{m_1}{m_2}\right) = 1680$$

$\Rightarrow v_1 = 12\ ms^{-1}$ and $\Rightarrow v_2 = -28\ ms^{-1}$ Thereby $v_1 - v_2 = [12-(-28)]ms^{-1} = 40\ ms^{-1}$

Thus we get $v_{rel} = 40\ ms^{-1}$

9. Two masses move on a collision path as shown. Before the collision the object with mass 2 M moves with a speed v making an angle $\theta = \sin^{-1}\frac{3}{5}$ to the x-axis while the object with mass M moves with a speed $\frac{3}{2}v$ making an angle $\phi = \sin^{-1}\frac{4}{5}$ with the x-axis. After the collision the object of mass 2 M is observed to be moving to the right along the x-axis with a speed of $\frac{4}{5}v$. There are no external forces acting during the collision. The correct option is **[NSEP-2021]**



(A) The velocity of mass M, after the collision, is zero.

(B) The centre of mass is moving along x-axis before the collision.

(C) The velocity of centre of mass after the collision is $\frac{5}{2}v$

(D) The total linear momentum of the system before the collision along x - axis is $\frac{5}{6}Mv$

Ans. (B)

Sol. By the principle of conservation of momentum along x-direction

$2Mv\cos\theta + M\frac{3}{2}v\cos\phi = 2M\times\frac{4}{5}v + MV_x$ where V_x is the velocity of mass M in x direction after the collision. Substituting the values $2Mv\times\frac{4}{5} + M\frac{3}{2}v\times\frac{3}{5} = 2M\times\frac{4}{5}v + MV_x \Rightarrow V_x = \frac{9}{10}v$ In $y-$ direction the momentum conservation yields $-2Mv\sin\theta + M\frac{3}{2}v\sin\phi = 2M\times 0 + MVy$ substituting the values $-2Mv\frac{3}{5} + M\times\frac{3}{2}v\times\frac{4}{5} = 0 + MV_y \Rightarrow V_y = 0$ means no velocity in y-direction. Hence the centre of mass moves along x-direction. The velocity of centre of mass after the collision is

$$V_{xCM} = \frac{2M\times\frac{4}{5}v + M\times\frac{9}{10}v}{3M} = \frac{25}{30}v = \frac{5}{6}v$$ and not $\frac{5}{2}v$ Also the linear momentum before collision is $\frac{25}{10}Mv = \frac{5}{2}Mv$ and not

10. Two small masses m and M lie on a large horizontal frictionless circular track of radius R. The two masses are free to slide on the track but constrained to move along a circle. Initially the two masses are tied by a thread with a compressed spring between them (spring of negligible length being attached with none of the two masses). The compressed spring stores a potential energy U_0. At a certain time $t=0$ the thread is burnt and the two masses are released to run opposite to each other leaving the spring behind. The total mechanical energy remaining conserved. On the circular track the two masses make a head on perfectly elastic collision. Take $M=2\,m$ for all calculations. Which of the following option(s) is / are correct? **[NSEP-2021]**

(A) The angle turned by mass m before the collision is $\theta=4\frac{\pi}{3}$

(B) The velocity of mass m on the track is $u=\sqrt{\frac{4U_0}{3m}}$

(C) The time taken to collide for the first time is $t_1=2\pi R\sqrt{\frac{m}{3U_0}}$

(D) The time taken for the second collision is $t_2=2\pi R\sqrt{\frac{2m}{3U_0}}$

Ans. **(ABC)**

Sol. When the masses are released, they move in opposite direction with equal momentum i.e.,

$$mv+MV=0\Rightarrow V=-\frac{mV}{M}\ldots\ (1)$$

Numerically $V=\frac{mV}{M}$

Let the two masses collide for the first time after time t and the mass m turns through angle θ during this period then $t=\frac{\theta}{\omega_1}=\frac{2\pi-\theta}{\omega_2}$ or

$$t=\frac{\theta R}{v}=\frac{(2\pi-\theta)R}{v}\Rightarrow\left(\frac{1}{v}+\frac{1}{v}\right)\theta=\frac{2\pi}{v}$$

Substituting the value of v,

we obtain $\theta=\frac{2\pi M}{m+M}=\frac{4\pi}{3}$

if $M=2m$.

Also Energy conservation provide that $\frac{1}{2}mv^2+\frac{1}{2}Mv^2=U_0$ or $\frac{1}{2}mv^2+\frac{1}{2}M\left(\frac{mv}{M}\right)^2=U_0$

$$\text{or } \frac{1}{2}mv^2\left[1+\frac{m}{M}\right]=U_0\Rightarrow v=\sqrt{\frac{2MU_0}{m(m+M)}}=\sqrt{\frac{4U_0}{3m}}$$

if $M=2m$ Thus the time taken for first collision is

$$t=\frac{\theta R}{v}=\frac{4\pi R}{3\sqrt{\frac{4U_0}{3\,m}}}=2\pi R\sqrt{\frac{m}{3U_0}}$$

Lastly the time taken for the second collision must be just double of it and not

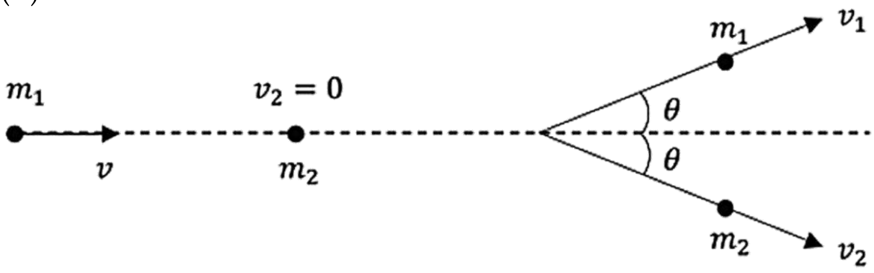
$$2\pi R\sqrt{\frac{2m}{3U_0}}$$

11. A ball A (mass m_1) moving with velocity v experiences an elastic collision with another stationary ball B (mass m_2). Each ball flies apart symmetrically relative to the initial direction of motion of ball A, at an angle θ. Ratio of the masses of balls $\frac{m_1}{m_2}$ is **[NSEP-2022]**

(A) $1+2\cos\theta$ (B) $2\cos2\theta$ (C) $1+2\cos2\theta$ (D) $1+\cos2\theta$

Ans. (C)

Sol.



In vertical direction $\Rightarrow m_1v_1\sin\theta = m_2v_2\sin\theta$

$\therefore m_1v_1 = m_2v_2$

$\therefore \frac{m_1}{m_2} = \frac{v_2}{v_1}$ ... (1)

In horizontal direction

$\Rightarrow m_1v + 0 = m_1v_1\cos\theta + m_2v_2\cos\theta$ ... (2)

Also K.E is conserved as the collision is elastic,

$\therefore \frac{1}{2}m_1v^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ ... (3)

From 1 & 2

$m_1v = 2m_1v_1\cos\theta \Rightarrow v_1 = \frac{v}{2\cos\theta}$ ... (4)

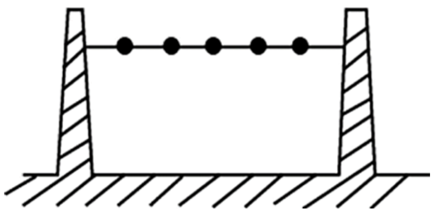
And $v_2 = \frac{m_1v_1}{m_2} \Rightarrow v_2 = \frac{m_1v}{2m_2\cos\theta}$

$\therefore$ om (3), (4) & (5)

$$\Rightarrow m_1v^2 = \frac{m_1v^2}{4\cos^2\theta} + \frac{m_2m_1^2v^2}{4m_2^2\cos^2\theta} \Rightarrow 1 = \frac{1}{4\cos^2\theta}\left[1+\frac{m_1}{m_2}\right] \Rightarrow \frac{m_1}{m_2} = 4\cos^2\theta - 1$$

$\therefore \frac{m_1}{m_2} = 1+2\cos2\theta$

12. A number n of identical balls, each of mass m and radius r are stringed like beads at random and at rest along smooth, rigid horizontal rod of length L mounted between immovable supports; $\frac{r}{L}$ is small but not negligible. **[NSEP-2022]**



Collision between balls, or between balls and supports, are perfectly elastic. One of the balls struct horizontally so as to acquire a speed v. Resulting outward force felt by supports, averaged over a long time, is

(A) $\frac{mv^2}{2(1-2nr)}$ (B) $\frac{mv^2}{(1-2nr)}$ (C) $\frac{2mv^2}{(L-2nr)}$ (D) $\frac{mv^2}{L}$

Ans. (B)

Sol. Total distance available for motion of beads $= L - 2nr$

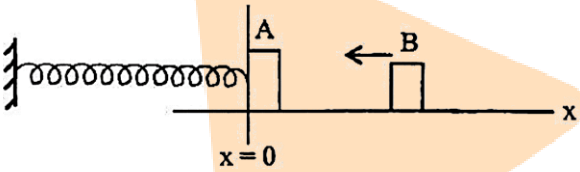
Change in momentum of a bead after collision $= 2mv$

Avg time b/w tow collision with one of the supports

$$t_{avg} = \frac{2(L-2nr)}{v}$$

$$F_{avg} = \frac{2mv}{\frac{2(L-2nr)}{v}} = \frac{mv^2}{L-2nr}$$

13. A small block A of mass 2 kg is attached to a spring of force constant $1200 Nm^{-1}$ and rests on a smooth , horizontal surface at $x=0$ as shown in figure. A second block B of mass 1 kg slides along the surface towards A at $6\ ms^{-1}$ and sticks to it. Assuming that the collision occurs at $t=0$, position x (in meter) of block A as a function of time t is expressed as **[NSEP-2022]**



(A) $x = 0.173\cos 20t$

(B) $x = 0.1\cos 40\pi t$

(C) $x = -0.173\sin\frac{\pi}{10}t$

(D) $x = 0.1\sin 20t$

Ans. (D)

Sol. $\omega = \sqrt{\frac{k}{(mA+ms)}} = \sqrt{\frac{1200}{2+1}} = 20 rad/s$

When B stick to A, $v_C = \frac{m_B / v_B}{m_A + m_B} = \frac{1\times 6}{2+1} = 2\ m/s$

We have, $V_c = A\omega$ where A is amplitude

$\Rightarrow 2 = A\times 20 \Rightarrow A = 0.1\ m$

As the combined mass is merging initially towards -ve x -axis, from $x=0$

$x = -A\sin\sin\omega t \Rightarrow x = -0.1\sin\sin 20t$

14. A deuteron of mass M moving at speed v collides elastically with an α - particle of mass $2M$, initially at rest. The deuteron is scattered through 90° from initial direction of its motion with speed V_d while the α-particle is scattered with speed V_α at an angle θ from the initial direction of motion of deuteron. Then **[NSEP-2023]**

(A) $\theta = 30^\circ$

(B) $V_a = \frac{V}{\sqrt{3}}$

(C) $V_d = \frac{V}{\sqrt{3}}$

(D) a fraction $\frac{2}{3}$ of energy of deuteron is transferred to α particle

Ans. (ABCD)

Sol. From LMC

In x direction, $MV = 2MV_{\alpha}\cos\theta$

$$\Rightarrow V_{\alpha}\cos\theta = \frac{V}{2} \quad ...(1)$$

In y direction, $MV_d = 2MV_{\alpha}\sin\alpha$

$$\Rightarrow V_{\alpha}\sin\theta = \frac{V_d}{2} \quad ...(2)$$

From K.E. conservation

$$\frac{1}{2}MV^2 = \frac{1}{2}MV_d^2 + \frac{1}{2}2M\left(V_{\alpha}^2\cos^2\theta + V_{\alpha}^2\sin^2\theta\right)$$

$$\frac{1}{2}MV^2 = \frac{1}{2}MV_d^2 + M\left(\frac{V^2}{4} + \frac{V_d^2}{4}\right)$$

$$\frac{MV^2}{4} = \frac{3MVd^2}{4}$$

$$\Rightarrow V_d = \frac{V}{\sqrt{3}}$$

From (1) and (2)

$$\tan\theta = \frac{V_d}{V} = \frac{1}{\sqrt{3}}$$

$$\theta = 30^{\circ}$$

$$\Rightarrow V_{\alpha} = V_d = \frac{V}{\sqrt{3}}$$

$$E_{\alpha} = \frac{1}{2}2MV_{\alpha}^2 = \frac{1}{2}2M\frac{V^2}{3}$$

$$E_{\alpha} = \frac{2}{3}\left(\frac{1}{2}MV^2\right)$$

$$E_{\alpha} = \frac{2}{3}E_d$$

RIGID BODY DYNAMICS

1. Consider two point masses m_1 and m_2 connected by a light rigid rod of length r_0. The moment of inertia of the system about an axis passing through their centre of mass and perpendicular to the rigid rod is given by **[NSEP-2017]**

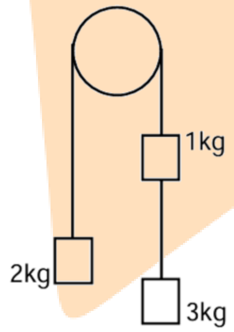
(A) $\frac{m_1m_2}{2(m_1+m_2)}r_0^2$ (B) $\frac{m_1m_2}{m_1+m_2}r_0^2$

(C) $\frac{2m_1m_2}{m_1+m_2}r_0^2$ (D) $\frac{m_1^2+m_2^2}{m_1+m_2}r_0^2$

Ans. (B)

Sol. $\mu r_0{}^2$

2. In the following arrangement the pulley is assumed to be light and the string inextensible. The acceleration of the system can be determined by considering conservation of a certain physical quantity. The physical quantity conserved and the acceleration respectively, are **[NSEP-2017]**



(A) Energy and $g/3$ (B) Linear momentum and $g/2$

(C) Angular momentum and $g/3$ (D) Mass and $g/2$

Ans. (A)

Sol. No dissipative forces so total mech energy is conserved

$$(3+1)gx-2gx=\frac{1}{2}(3+1+2)v^2$$

$$\Rightarrow 2gx=3v^2$$

$$\Rightarrow 2gv=6v\frac{dv}{2dt}$$

$$\Rightarrow \frac{dv}{dt}=\frac{g}{3}$$

3. A uniform solid drum of radius R and mass M rolls without slipping down a plane inclined at an angle θ. Its acceleration along the plane is **[NSEP-2018]**

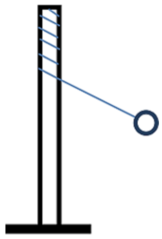
(A) $\frac{1}{3}g\sin\theta$ (B) $\frac{1}{2}g\sin\theta$ (C) $\frac{2}{3}g\sin\theta$ (D) $\frac{5}{7}g\sin\theta$

Ans. (C)

Sol. $a=\frac{g\sin\theta}{1+(k^2/R^2)}$

$$=\frac{g\sin\theta}{1+1/2}=\frac{2g\sin\theta}{3}$$

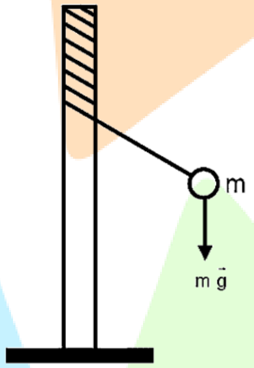
4. A string of length ℓ, tied to the top of a pole, carries a ball at its other end as shown. On giving the ball a single hand blow perpendicular to the string, it acquires an initial velocity v_0 in the horizontal plane and moves in a spiral of decreasing radius by curling itself around the pole. Therefore, **[NSEP-2018]**



(A) The instantaneous centre of revolution of the ball is the point of contact of the string with the pole at that instant.
(B) The instantaneous centre of revolution of the ball will be fixed at the point where the string as initially fixed.
(C) The angular momentum of the system will not be conserved.
(D) The angular momentum of the system will be conserved.

Ans. (AC)

Sol. Torque of $m\vec{g}$ is not zero m the plane perpendicular to it. Therefore angular momentum component in that plane is not conserved.
The Instantaneous axis revolution is instantaneously at rest.
$\therefore A$ is correct



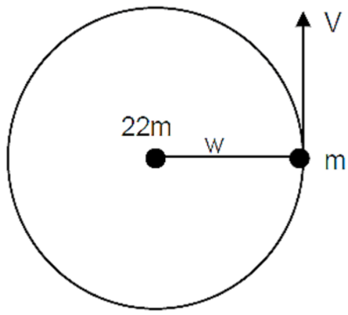
5. A large horizontal uniform disc can rotate freely about a rigid vertical axis passing through its centre O. A man stands at rest at the edge of the disc at a point A. The mass of the disc is 22 times the mass of the man. The man starts moving along the edge of the disc. When he reaches A, after completing one rotation relative to the disc, the disc has turned through.

[NSEP-2019]

(A) 30° (B) 90° (C) 60° (D) 45°

Ans. (A)

Sol. v : vel of man wrt disc



$$m(v-\omega R)R = \frac{22mR^2\omega}{2}$$

$$v-\omega R = 11\omega R$$

$$\omega R = \frac{V}{12}$$

$$t = \frac{2\pi R}{V}$$

$$\theta = \omega t = \omega\frac{2\pi R}{v} = \frac{2\pi}{12} = \frac{\pi}{6} = 30^\circ$$

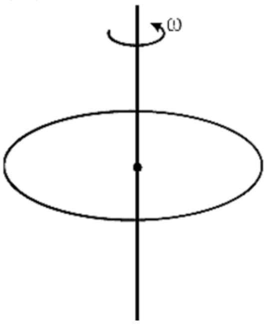
6. A uniform circular disc rotating at a fixed angular velocity ω about an axis normal to its plane and passing through its centre has kinetic energy E. If the same disc rotates with an angular velocity 2ω about a parallel axis passing through the edge, its kinetic energy will be:

[NSEP-2019]

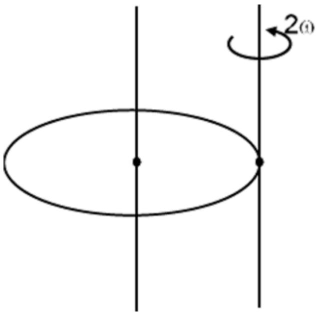
(A) 2 E (B) 4 E (C) 10 E (D) 12 E

Ans. (D)

Sol.



$$E = \frac{1}{2} I\omega^2$$



$$E' = \frac{1}{2}(I')(2\omega)^2$$

$$\frac{E'}{E} = \frac{\frac{1}{2}\left(\frac{3}{2}mR^2\right)4\omega^2}{\frac{1}{2}\frac{mR^2}{2}\omega^2}$$

$$E' = 12E$$

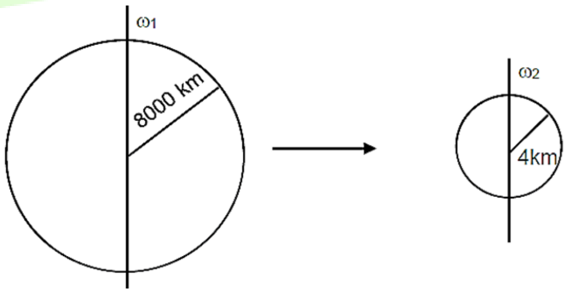
7. A star undergoes a supernova explosion. Just after the explosion, the material left behind forms a uniform sphere of radius 8000 km with a rotation period of 15 hours. This remaining material eventually collapses into a neutron star of radius 4 km with a period of rotation.

[NSEP-2019]

(A) 14 s (B) 3.8 h (C) 0.021 s (D) 0.0135 s

Ans. (D)

Sol. $L_i = L_f$



$$I_1\omega_1 = I_2\omega_2$$

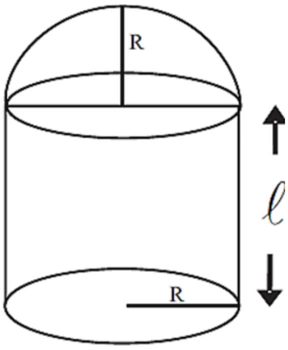
$$\Rightarrow \frac{2}{5}mR_1^2\omega_1 = \frac{2}{5}mR_2^2\omega_2$$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)^2$$

$$\Rightarrow T_2 = \left(\frac{R_2}{R_1}\right)^2 \times T_1$$

$$= \left(\frac{4}{28000}\right)^2 \times 15 \times 60 \times 60 = 0.013500$$

8. A solid hemisphere is cemented on the flat surface of a solid cylinder of same radius R and same material. The composite body is rotating about the axis of the cylinder of length ℓ with angular speed ω. The radius of gyration K is **[NSEP-2019]**



(A) $R\sqrt{\frac{2}{5}\left(\frac{15R+8\ell}{3R+2\ell}\right)}$ (B) $R\sqrt{\frac{1}{10}\left(\frac{15\ell+8R}{3\ell+2R}\right)}$ (C) $R\sqrt{\frac{3}{10}\left(\frac{15R+8\ell}{3R+2\ell}\right)}$ (D) $R\sqrt{\frac{1}{10}\left(\frac{3\ell+2R}{15\ell+8R}\right)}$

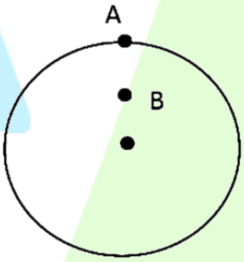
Ans. (B)

Sol. The mass of the composite system is $M=\frac{2}{3}\pi R^3\rho+\pi R^2\ell\rho=\frac{2}{3}\pi R^3\rho\left(1+\frac{3\ell}{2R}\right)$

The moment of Inertia is $I=\frac{2}{5}\times\frac{2}{3}\pi R^3\rho R^2+\frac{1}{2}\pi R^2\ell\rho R^2=\frac{4}{15}\pi R^5\rho\left(1+\frac{15\ell}{8R}\right)$

using now $I=MK^2$ we get $K=\sqrt{\dfrac{\frac{4}{15}\pi R^5\rho\left(1+\frac{15\ell}{8R}\right)}{\frac{2}{3}\pi R^3\rho\left(1+\frac{3\ell}{2R}\right)}}=R\sqrt{\dfrac{\frac{1}{1}\left(\frac{(8R+15\ell)}{10}\right)(2R+3\ell)}{(2R+3\ell)}}$

9. A circular disc of radius $R=10$ cm is uniformly rolling on a horizontal surface with a velocity $v=4\ \text{ms}^{-1}$ of centre of mass without slipping, the time taken by the disc to have the speed of point A (which lies on the circumference) equal to the present speed of point B (point B lies midway between centre and the point A) is **[NSEP-2021]**



(A) $t=0.025$ s (B) $t=0.036$ s (C) $t=0.046$ s (D) $t=0.064$ s

Ans. (B)

Sol. Under the conditions of pure rolling of the disc, the velocity of the point A (at the top) on the circumference is $v+\omega R=2v$ where as the velocity of point B at half the radius is $v+\omega\frac{R}{2}=\frac{3}{2}v$

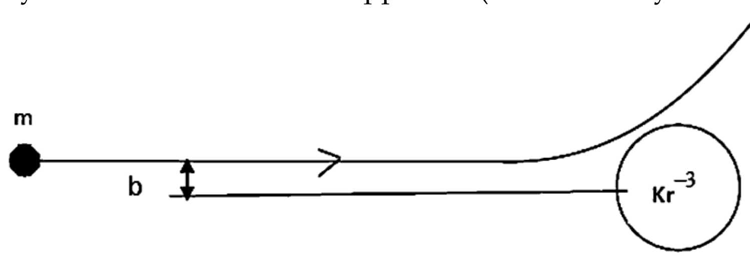
Let the final speed of point A becomes $\frac{3}{2}v$ after turning through an angle ϕ then

$\frac{3}{2}v=\sqrt{v^2+\omega^2R^2+2v\omega R\cos\phi}=\sqrt{v^2+v^2+2v^2\cos\phi}\Rightarrow\frac{3}{2}=\sqrt{2+2\cos\phi}$ or

$\Rightarrow\frac{3}{2}=\sqrt{2+2\left(2\cos^2\frac{\phi}{2}-1\right)}=2\cos\frac{\phi}{2}$ or $\cos\frac{\phi}{2}=\frac{3}{4}\Rightarrow\phi=82.82^\circ$

Further if $T=\frac{2\pi R}{v}$ be the time period then by simple unitary method time taken to turn through $\phi=82.82^\circ$ is $t=\frac{\phi}{360}\times\frac{2\pi R}{v}=0.036$ s

10. As shown in the figure, a particle of mass $m = 10^{-10}$ kg, moving with velocity $v_0 = 10^5$ m/s approaches a stationary fixed target with impact parameter b from a large distance. If the fixed rigid target has a core with repulsive central force $F(r) = \frac{K}{r^3}$ where constant $K > 0$ and the particle scatters elastically. The closest distance of approach (if numerically $K = b^2$) is **[NSEP-2021]**



(A) b (B) $b\sqrt{2}$ (C) $b\sqrt{3}$ (D) 2 b

Ans. (B)

Sol. At the point of closest approach (distance) the particle will have tangential velocity expressed as $v_t = \omega d = \dot{\theta} d$ By conservation of angular momentum

$$mvb = I\omega = md^2\dot{\theta} \Rightarrow \dot{\theta} = \omega = \frac{vb}{d^2}$$

This being a case of elastic scattering, the conservation of energy provides

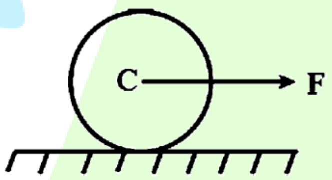
$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}md^2\omega^2 - \int \frac{K}{r^3} \cdot dr \Rightarrow mv^2 = md^2\omega^2 + \frac{K}{d^2} \because PE = -\int \frac{K}{r^3} dr = \frac{K}{2r^2} = \frac{K}{2d^2}$$

Thereby $mv^2 = md^2\left(\frac{vb}{d^2}\right)^2 + \frac{K}{d^2} = \left(mv^2b^2 + K\right)\frac{1}{d^2}$

Substituting $m = 10^{-10}$Kg, $v = 10^5 ms^{-1}$ and numerically $K = b^2$

we obtain $d^2 = \left(mv^2b^2 + k\right)\frac{1}{mv^2} = 2b^2 \Rightarrow d = b\sqrt{2}$

11. A solid cylinder of mass m is rolling without sliping on a rough horizontal surface, under the action of horizontal force F such that the line of action of F passes through centre C of the cylinder. Choose the correct alternative. **[NSEP-2022]**



(A) Acceleration of centre of cylinder is $\frac{F}{m}$ (B) Frictional force on cylinder acts forward

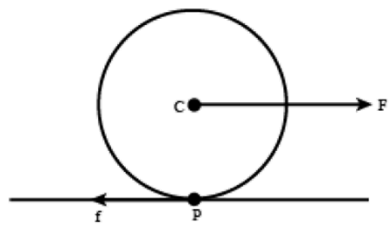
(C) Magnitude of friction force is $\frac{F}{3}$ (D) None of the above.

Ans. (C)

Sol. Let's calculate Torque about C,

$\tau = I\alpha$

Assuming radius of cylinder to be R,



$$Rf = \left(\frac{1}{2}mR^2\right)\alpha$$

$$f = \frac{1}{2}mR\alpha \left(\because a = R\alpha\right)$$

$$\Rightarrow f = \frac{1}{2}ma \quad \ldots (1)$$

In translatory motion,

$F_{net} = ma$

$F - f = ma$

From eqn. (1)

$F - \frac{1}{2}ma = ma$

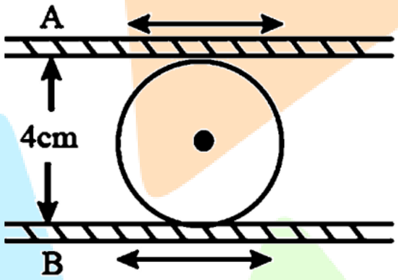
$F = \frac{3}{2}ma$

$\Rightarrow a = \frac{2\,F}{3\,m}$

$\Rightarrow$ friction force, $f = \frac{1}{3}F$

$\Rightarrow$ friction acts backwards.

12. In a certain machine two steel plates are separated by a hardened steel cylindrical roller (see fig). In operation, the plates move back and forth horizontally, perpendicular to the axis of roller, and the roller rolls freely between plates without slipping on either one. At a particular instant plate A is moving with a speed of 18 cmsec^{-1} to the right and an acceleration of 30 cm sec^{-2} to the left, and the plate B is moving with a speed of $6\ \text{cmsec}^{-1}$ to the to the right and an acceleration of $8\ \text{cmsec}^{-2}$ to the left. At that instant, for the roller **[NSEP-2022]**



(A) Its angular speed is 3 rad sec^{-1} clockwise

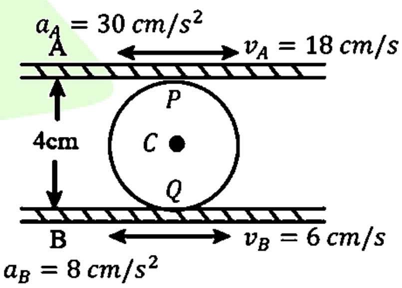
(B) Its angular acceleration is 6 rad sec^{-2} clockwise

(C) The linear speed of its axis is $12\ \text{cmsec}^{-1}$ towards right

(D) The linear acceleration of its axis is $20\ \text{cmsec}^{-2}$ towards left

Ans. **(AC)**

Sol. $a_B = 8\ cm/s^2$



$\vec{V}_p = 18\hat{i}$

$\vec{V}_Q = 6\hat{i}$

Let the velocity of C is $V_C\hat{i}$ and angular velocity about C is

$\omega\left(-\hat{k}\right)$

From the condition of pure rolling,

$v_p = v_c + \omega R$

$v_Q = v_c - \omega R$

$R = 2$ cm (given)

$V_C + 2\omega = 18$

$V_c - 2\omega = 6$

$\Rightarrow v_c = 12\frac{cm}{s}; \omega = 3rad/s$ (clockwise)

Let the acceleration of C is $a_c\hat{i}$ and angular acceleration about C is $\propto\left(-\hat{k}\right)$

$\vec{a}_{P_t}$ (tangential acceleration of p) $= \vec{a}_A$

$\vec{a}_{Q_t}$ (tangential acceleration of Q) $= \vec{a}_B$

$a_{P_t} = a_c + R\alpha$

$a_{Q_t} = a_c - R\alpha$

$a_c + 2\propto = -30$

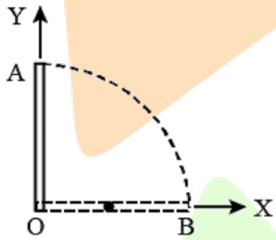
$a_c - 2\propto = -8$

$\Rightarrow a_c = 19\left(-\hat{i}\right) cm/s^2$

$\propto = \frac{11}{2}\left(\hat{k}\right) cm/s^2$

Correct option: (A), (C)

13. A long uniform rod of length L and mass M is pivoted vertically on a horizontal, friction less pivot at its lower end. The rod is released from rest in its vertical position OA (see figure). It falls off without slipping at O. At the instant the rod is horizontal, **[NSEP-2022]**



(A) Its angular speed is $\sqrt{\frac{3g}{L}}$

(B) Magnitude of its angular acceleration is $\frac{3g}{2L}$

(C) Acceleration of its centre of mass $\vec{a}_{cm} = -\frac{3g}{4}\hat{j}$($\hat{j}$ unit vector in Y direction)

(D) Reaction force at pivot $\frac{Mg}{4}\hat{j}$ (Take X, Y axis as shown)

Ans. (AB)

Sol. According to conservation of total mechanical energy at vertical and horizontal position

$Mgh = \frac{1}{2}|\omega^2$

$\frac{MgL}{2} = \frac{1}{2}\frac{ML^2}{3}\omega^2$

$\Rightarrow \omega^2 = \frac{3g}{L}$

$\Rightarrow \omega = \sqrt{\frac{3g}{L}}$

$\therefore$ Option A is correct.

Torque about 0 at horizontal position

$$mg\frac{L}{2}\left(-\hat{k}\right)=I\vec{\alpha}$$

$$Mg\frac{L}{2}\left(-\hat{k}\right)=\frac{ML^2}{3}\vec{\alpha}\Rightarrow|\alpha|=\frac{3g}{2L}$$

$\therefore$ Option B is correct.

Centre of mass has two acceleration components. One in downward direction and other towards centre.

$$a_y=\alpha\frac{L}{2}=\frac{3g}{4}$$

$$a_x=\omega^2\frac{L}{2}\left(-\hat{i}\right)\ \text{(centripetal acceleration)}$$

$\therefore$ Option C is incorrect.

Taking two component of reaction force at point

$$Mg-F_y=M_{ay}$$

$$F_y=Mg-ma_y$$

$$F_y=mg-\frac{3mg}{4}=\frac{mg}{4}$$

$$\vec{F}_y=\frac{mg}{4}\hat{j}$$

$$F_x=M\omega^2\frac{L}{2}$$

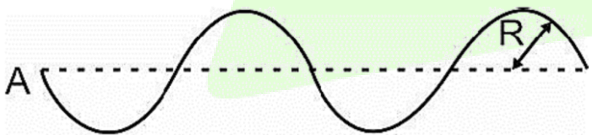
$$F_x=M\frac{3g}{L}\times\frac{L}{2}=\frac{3Mg}{2}$$

$$F_x=\frac{3Mg}{2}\left(-\hat{i}\right)$$

$\therefore$ Option D is incorrect.

Correct option (A, B)

14. A thin uniform rod of mass M is bent into four adjacent semicircles of radius of curvature R lying in same plane. Moment of inertia of the bent rod about an axis through one end A and perpendicular to plane of the rod is **[NSEP-2022]**



(A) $\frac{17}{2}MR^2$ (B) $44MR^2$ (C) $22MR^2$ (D) $\frac{43}{2}MR^2$

Ans. (C)

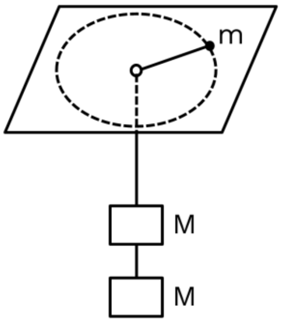
Sol. If we complete all the semi-circular rings into a complete ring then moment of inertia about given axis will be

$$I=\left(\frac{M}{2}R^2+\frac{M}{2}R^2\right)+\left(\frac{M}{2}R^2+\frac{M}{2}(3R)^2\right)+\left(\frac{M}{2}R^2+\frac{M}{2}(5R)^2\right)+\left(\frac{M}{2}R^2+\frac{M}{2}(7R)^2\right)=44MR^2$$

All the rings are half so desired moment of inertia is

$$I_0=\frac{I}{2}=22MR^2$$

15. A particle of mass m is revolving in a horizontal circle on a frictionless horizontal table with the help of a string tied to it and passing through a hole at the center of the table. Two equal masses M are attached to the other end of the string as shown. If one of the hanging masses M is removed gently, the radius of the circular motion of m **[NSEP-2023]**



(A) decreases by a factor 1.414
(B) increases by a factor 1.260
(C) increases by a factor 1.414
(D) does not change because of the conservation of angular momentum.

Ans. (B)

Sol. $2Mg = \frac{mV_1^2}{R_1}$

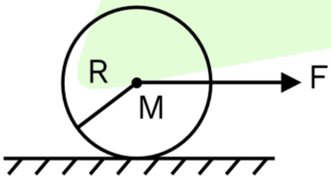
$Mg = \frac{mV_2^2}{R_2}$

$2 = \left(\frac{V_1}{V_2}\right)^2 \times \frac{R_2}{R_1}$

$2 = \left(\frac{R_2}{R_1}\right)^3$

$(2)^{1/3} = \frac{R_2}{R_1}$

16. A lawn roller is a solid cylinder of mass M and radius R. As shown in the figure, it is pulled at its center by a horizontal force F and rolls without slipping on a horizontal surface. Then the **[NSEP-2023]**



(A) Acceleration of the cylinder is $\frac{2F}{M}$

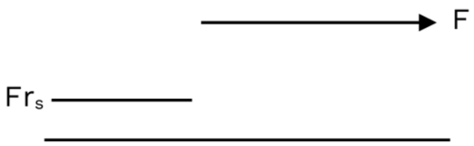
(B) Force of friction acting on the cylinder is $\frac{2F}{3M}$

(C) Coefficient of friction needed to prevent slipping is at least $\frac{F}{3Mg}$

(D) Minimum coefficient of friction to prevent slipping is $\frac{2F}{3Mg}$

Ans. (C)

Sol.



$F - Fr = M\alpha$

$Fr + R = \frac{MR^2}{2}\alpha$

$F = \frac{3M\alpha}{2}$

$Fr = \frac{MR\alpha}{2}$

$\alpha = \frac{2F}{3M}$

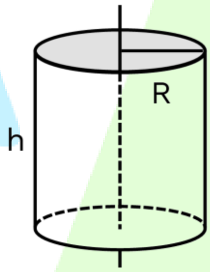
$Fr = \frac{M\alpha}{2} = \frac{M}{2} \times \frac{2F}{3M} = \frac{F}{3}$

$\frac{F}{3} \leq \mu mg$

$\mu \leq \frac{F}{3Mg}$

$\mu_{min} = \frac{F}{3Mg}$

17. A can is a hollow cylinder of radius R and height h. Its ends are sealed with circular sheets of the same material. The can is made of thin sheet metal of areal mass density $\sigma(kg/m^2)$. Moment of inertia of this closed can about its vertical axis of symmetry is **[NSEP-2023]**



(A) $\pi R^3\sigma(h+2R)$ (B) $\pi R^3\sigma(h+R)$

(C) $\pi R^3\sigma(2h+R)$ (D) $2\pi R^3\sigma(h+R)$

Ans. (C)

Sol. Moment of inertia of can is summation of moments of inertia of 2 disc & one hollow cylinder about the same axis.

$\therefore I = 2\left(\frac{m_D R^2}{2}\right) + m_c R^2 \quad \ldots\ldots\ldots(1)$

Now $m_D = \sigma.\pi R^2$ & $m_C = \sigma.2\pi Rh$

$\therefore I = \sigma\pi R^4 + 2\sigma\pi R^3\ h$

$\therefore I = \sigma\pi R^3(R + 2\ h)$